A Single-Mesh Method for Electromagnetic Field and Thermal Field Coupled Finite-Element Analysis

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This paper presents a single-mesh method for electromagnetic field and thermal field coupled finite-element analysis. In the proposed adaptive degrees-of-freedom (DoF) algorithm, only a single mesh is required for both electromagnetic field computation and thermal field computation, while the master-slave technique is adopted to coarsen the distribution of computational DoFs according to the estimated solution errors. This progress can avoid solution interpolation errors due to the changes between the mesh for electromagnetic field computation and the mesh for the thermal field computation. In the meantime, the posteriori error estimator is applied to ensure the accuracies of the solutions. The details of the algorithm are presented and a numerical example using the proposed algorithm is shown to verify its validity and effectiveness in the electromagnetic-thermal co-simulation.

***Index Terms*—Adaptive mesh method, degrees-of-freedom, electromagnetic field, finite element method, thermal field.**

I. Introduction

THE electromagnetic field and thermal field coupled finite-element method (FEM) is widely used in electrical engineering nowadays, especially in electric machine design and optimization. Generally, the electromagnetic field and thermal field is indirectly coupled. Namely, the losses of electromagnetic field are applied to the thermal field as a source. As the gradient distributions of electromagnetic field and thermal field are different, the refined areas of the meshes will be different for the two fields. Therefore, two different meshes are required for the two different field computations. The data between the meshes need to be mapped each other, which will produce additional numerical error and increase computing time.

To solve this problem and optimize the electromagnetic field and thermal field coupled analysis, this paper proposes an adaptive degree-of-freedom (DoF) finite-element algorithm. The advantage of the proposed method is that only single dense FEM mesh is required, and the accuracies of the both fields calculation can be ensured. The slave-master technique [1] is adopted to coarsen the computational DoFs according to the estimated solution errors. The DoFs are removed from the areas if the errors are small enough [2].

II. Basic System Equations

For simplicity, the discussion of the principle supporting the proposed algorithm will be limited to a two-dimensional (2-D) problem. During the electromagnetic field and thermal field coupled analysis, the basic equations of electromagnetic field eddy-current problem are summarized as follows:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial \dot{A}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial \dot{A}}{\partial y}\right)-j\omega\sigma\dot{A}=-\dot{J}_s \tag{1}$$

where $\dot{A}$ is the z-component of the magnetic vector potential; μ is the magnetic permeability of material; σ is the conductivity of material and $\dot{J}_s$ is the current density of excitation.

The eddy-current loss distributions per unit volume q can be obtained according to the results of the electromagnetic field computation:

$$q=\int\frac{J^2}{\sigma}dS \tag{2}$$

$$\dot{J}=\dot{J}_e+\dot{J}_s \tag{3}$$

where $\dot{J}$ is the total current density and $\dot{J}_e=-j\omega\sigma\dot{A}$ is the eddy-current density.

Then these losses will be adopted as the heat source and the temperature distribution can be computed based on the heat conduction equation [3]:

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+q-\rho c\frac{\partial T}{\partial t}=0 \tag{4}$$

where k is the thermal conductivity, ρ is the density, c is the heat capacity and q is the internal heat source. The convective boundary condition is applied as follows:

$$-k\frac{\partial T}{\partial n}|_\Gamma=\alpha(T-T_f)|_\Gamma \tag{5}$$

where T and T_f are the boundary and air temperature, respectively; and α denotes the heat transfer coefficient.

III. Adaptive Degrees-of-Freedom Algorithm

The proposed adaptive DoF algorithm mainly involves DoF coarsening. For the DoF coarsening method, the constraint equations for the DoF to be removed are introduced, and they can be obtained by master-slave method [1]. Specifically, each DoF to be removed is a slave node; the value of the slave DoF is depending on the values of its master DoFs and hence is not need to be solved. Meanwhile, the symmetry property of the original system will be upheld, and the unknowns of the system will be reduced.

To ensure the master-slave constraint equations assembled into the global matrix conveniently, it's important to build the data structure of the mesh node. The data members in a DoF class consist of the nodes' coordinates, the DoF ID, the flag which indicates whether it is a master node or slave node, the global unknown ID in the matrix equation. If it is a slave DoF, it should also include its master DoF IDs and the weighting values with the master DoFs [5].

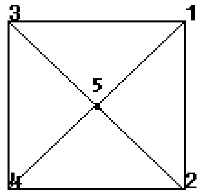

Fig. 1. A simple mesh to demonstrate the master node and slave node.

For example, in Fig. 1, if the node with ID=5 is a slave node, and its solution is dependent on the nodes with ID=2, 3, the constraint equation

$$s_5 = m_{25} \cdot s_2 + m_{35} \cdot s_3 \tag{6}$$

can be substituted into the global matrix equation to eliminate unknowns s_5 .

Assemble the transformation into the global finite element matrix equation (7) :

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{pmatrix} \tag{7}$$

and it can be modified as follows:

$$\begin{bmatrix} a_{11} & a_{12} + m_{25} \cdot a_{15} & a_{13} + m_{35} \cdot a_{15} & a_{14} \\ a_{21} & a_{22} + m_{25} \cdot a_{25} & a_{23} + m_{35} \cdot a_{25} & a_{24} \\ a_{31} & a_{32} + m_{25} \cdot a_{35} & a_{33} + m_{35} \cdot a_{35} & a_{34} \\ a_{41} & a_{42} + m_{25} \cdot a_{45} & a_{43} + m_{35} \cdot a_{45} & a_{44} \end{bmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} \tag{8}$$

Assemble the transformation into the global finite element matrix equation and solve it, then the solutions of the slave DoFs can be recovered according to the constraint equations.

IV. Adaptive Degrees-of-Freedom Algorithm

To verify the adaptive DoF algorithm for electromagnetic field and thermal field coupled finite element analysis, the temperature rise in a motor slot with a single layer winding shown in Fig. 2 is to be solved.

During the analysis process, only one mesh containing 3713 nodes is used and the adaptive DoF algorithm is applied for both electromagnetic field computation and thermal field computation. The FEM mesh, the initial numerical solution and the numerical solution using the adaptive DoF algorithm of both the magnetic potential and the temperature rise are shown in Fig. 3(a)–(e), respectively.

The numerical experiment results show that the proposed algorithm can ensure the accuracy of both fields calculation using a single mesh. It's also valid for the solutions of multi-physics. For the electromagnetic field and thermal field coupled analysis, there are about 10% and 8% slave DoFs for electromagnetic field and thermal field, respectively, and the computational time is saved obviously, comparing with traditional algorithm.

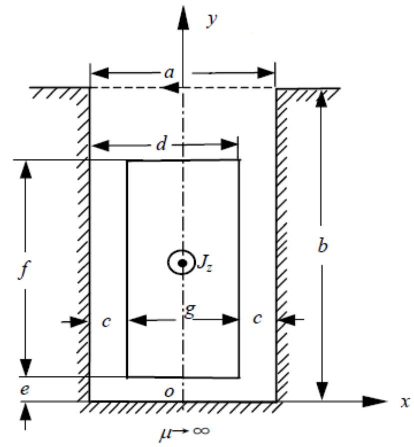

Fig. 2. The sectional view of a motor slot. a =0.8cm, b =1.6 cm, c =0.1 cm, d =0.7 cm, e =0.1 cm, f =1.2 cm, g =0.6 cm, J_z =250A/cm^2, and the excitation frequency is 50Hz.

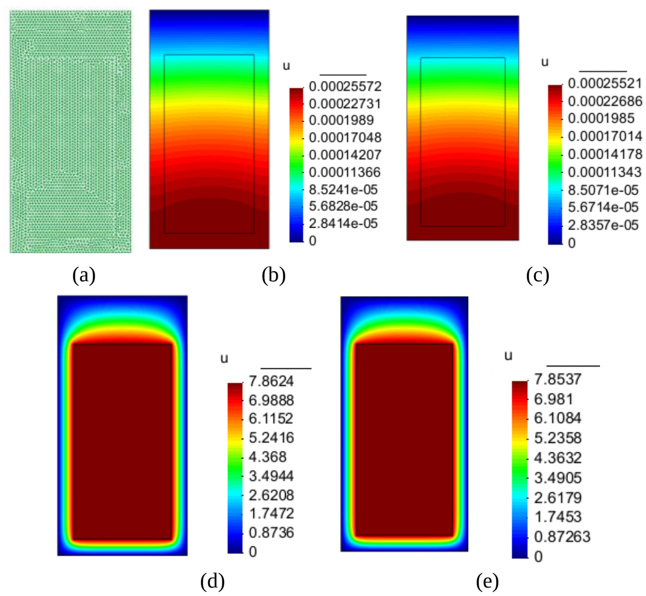

Fig. 3. (a) The FEM computational mesh. (b) The initial numerical solution of electromagnetic field. (c)The numerical solution using the adaptive DoF algorithm of electromagnetic field. (d) The initial numerical solution of thermal field. (e) The numerical solution using the adaptive DoF algorithm of thermal field.

V. References

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